UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL

Semestre: 2012 – 2 ***TIPO "A"***
31 de mayo de 2012

INSTRUCCIONES: Lea cuidadosamente los enunciados de los 7 reactivos que componen el examen antes de empezar a resolverlos. La duración máxima del examen es de **2.5 horas**.

Nombre del alumno: ____ No. de cuenta: ______________________

1) Determine la solución en forma explícita de la siguiente ecuación diferencial

$$x\,y^2\,y' - y^3 = 1$$

10 PUNTOS

2) Resuelva la ecuación diferencial

$$y'' + y' - 2\,y = 10\,sen(3x)$$

15 PUNTOS

3) Resuelva, por el método de variación de parámetros, la ecuación diferencial

$$y'' - 16\,y = \frac{16\,x}{e^{4x}}$$

15 PUNTOS

4) Determine en la ecuación diferencial $y'' + 4\,y' + 4\,y = 8$ su sistema equivalente de ecuaciones diferenciales de primer orden y resuelva además uno de sus valores.

15 PUNTOS

5) Resuelva el problema de valor inicial

$$x'' - x' - 6\,x = 60\,u(t - \pi) \quad ; \quad x(0) = 0 \ , \ x'(0) = 0$$

15 PUNTOS

6) Resuelva la ecuación integro-diferencial $\frac{dy}{dt} = -\,sen(t) - \int_0^t y(\tau)\,d\tau$ para $y(0) = 1$,

considerando que $sent * \cos t = \frac{1}{2} t\,sent$

15 PUNTOS

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7) Resuelva la ecuación diferencial en derivadas parciales

$$\frac{\partial^2 u}{\partial x^2} - 3\frac{\partial u}{\partial y} = 0$$

suponga una constante de separación igual a 3.

15 PUNTOS

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL
SEMESTRE 2012-2 TIPO A
PROFRA:ING RAQUEL MARTÍNEZ AVALOS

REACTIVO 1
Ecuación diferencial de primer orden no lineal

> $Ecuacion := x \cdot y \cdot \cdot 2 \cdot diff(y(x), x) - y \cdot \cdot 3 = 1$

$$Ecuacion := x\, y^2 \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - y^3 = 1 \quad \mathbf{(1)}$$

> $EcuacionNormalizada := lhs(Ecuacion) \cdot \left(\frac{1}{x\, y^2}\right)$

$$EcuacionNormalizada := \frac{x\, y^2 \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - y^3}{x\, y^2} \quad \mathbf{(2)}$$

> $simplify\left(\frac{simplify\left(numer\left(\frac{x\, y^2 \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - y^3}{x\, y^2}\right), symbolic\right)}{simplify\left(denom\left(\frac{x\, y^2 \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - y^3}{x\, y^2}\right), symbolic\right)}\right)$

$$\frac{x \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - y}{x} \quad \mathbf{(3)}$$

> $EcuacionNormalizada2 := diff(y(x), x) - \left(\frac{y(x)}{x}\right) = \frac{1}{x \cdot y(x) \cdot \cdot 2}$

$$EcuacionNormalizada2 := \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) - \frac{y(x)}{x} = \frac{1}{x\, y(x)^2} \quad \mathbf{(4)}$$

> with(DEtools) :

> odeadvisor(EcuacionNormalizada2);

[_separable] **(5)**

> restart

>

> $M_{xy} := factor\left(-\frac{2\, y}{x} - \frac{1}{x \cdot y \cdot \cdot 2}\right)$

$$M_{xy} := -\frac{2\, y^3 + 1}{x\, y^2} \quad \mathbf{(6)}$$

> $M := \left(-\frac{1}{x}\right) \cdot \left(\frac{2\, y^3 + 1}{y^2}\right)$

(7)

$$M := -\frac{2y^3+1}{x y^2} \quad \textbf{(7)}$$

> $N := 1$

$$N := 1 \quad \textbf{(8)}$$

> $P(x) := -\left(\frac{1}{x}\right)$

$$P(x) := -\frac{1}{x} \quad \textbf{(9)}$$

> $Q(y) := -\frac{y\cdot\cdot 2}{2y^3+1}$

$$Q(y) := -\frac{y^2}{2y^3+1} \quad \textbf{(10)}$$

> $SolucionGeneral := int(P(x), x) + int(Q(y), y) = C1$

$$SolucionGeneral := -\ln(x) - \frac{1}{6}\ln\left(2y^3+1\right) = C1 \quad \textbf{(11)}$$

2 REACTIVO

> $restart$

> $EcuacionLineal := diff(y(x), x\$2) + diff(y(x), x) - 2\,y(x) = 10\sin(3\cdot x)$

$$EcuacionLineal := \frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) - 2\,y(x) = 10\sin(3x) \quad \textbf{(12)}$$

> $EcuacionHomogenea := lhs(EcuacionLineal) = 0$

$$EcuacionHomogenea := \frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) - 2\,y(x) = 0 \quad \textbf{(13)}$$

> $EcuacionCaracteristica := m\cdot\cdot 2 + m - 2 = 0$

$$EcuacionCaracteristica := m^2 + m - 2 = 0 \quad \textbf{(14)}$$

> $Raiz := solve(EcuacionCaracteristica)$

$$Raiz := 1, -2 \quad \textbf{(15)}$$

> $Sol_1 := y(x) = \exp\left(Raiz_1\cdot x\right); Sol_2 := y(x) = \exp\left(Raiz_2\cdot x\right)$

$$Sol_1 := y(x) = e^x$$

$$Sol_2 := y(x) = e^{-2x} \quad \textbf{(16)}$$

> $SolucionHomogenea := y(x) = C1\cdot rhs\left(Sol_1\right) + C2\cdot rhs\left(Sol_2\right)$

$$SolucionHomogenea := y(x) = C1\,e^x + C2\,e^{-2x} \quad \textbf{(17)}$$

> $Q(x) := rhs(EcuacionLineal)$

$$Q(x) := 10\sin(3x) \quad \textbf{(18)}$$

> $OperadorAnulador := (D\cdot\cdot 2 + 9)$

$$OperadorAnulador := D^2 + 9 \quad \textbf{(19)}$$

> $RaizAnulador := solve(m\cdot\cdot 2 + 9)$

$$RaizAnulador := 3\,I, -3\,I \quad \textbf{(20)}$$

> $FormaSolPart := C3\cdot\cos(3\cdot x) + C4\cdot\sin(3\cdot x)$

(21)

$$FormaSolPart := C3\cos(3x) + C4\sin(3x) \quad (21)$$

> Sistema := diff(y(x), x) = diff(FormaSolPart, x), diff(y(x), x$2) = diff(FormaSolPart, x$2) : Sistema[1]; Sistema[2];

$$\frac{d}{dx}y(x) = -3\,C3\sin(3x) + 3\,C4\cos(3x)$$

$$\frac{d^2}{dx^2}y(x) = -9\,C3\cos(3x) - 9\,C4\sin(3x) \quad (22)$$

>

> EcuacionSust := rhs(Sistema[1]) + rhs(Sistema[2]) − 2·FormaSolPart = 10 sin(3·x)

$$EcuacionSust := -3\,C3\sin(3x) + 3\,C4\cos(3x) - 11\,C3\cos(3x) - 11\,C4\sin(3x) = 10\sin(3x) \quad (23)$$

> SolParaC1 := −3 C3 sin(3 x) − 11 C4 sin(3 x) = 10 sin(3 x)

$$SolParaC1 := -3\,C3\sin(3x) - 11\,C4\sin(3x) = 10\sin(3x) \quad (24)$$

> Ecuacion1C := −3 C3 − 11 C4 = 10

$$Ecuacion1C := -3\,C3 - 11\,C4 = 10 \quad (25)$$

> Ecuacion2C := 3 C4 − 11 C3 = 0

$$Ecuacion2C := 3\,C4 - 11\,C3 = 0 \quad (26)$$

> Sistema := Ecuacion1C, Ecuacion2C

$$Sistema := -3\,C3 - 11\,C4 = 10,\ 3\,C4 - 11\,C3 = 0 \quad (27)$$

> ValordeCtes := solve({Sistema}, {C3, C4});

$$ValordeCtes := \left\{C3 = -\frac{3}{13},\ C4 = -\frac{11}{13}\right\} \quad (28)$$

> SolucionParticular := subs(C3 = −3/13, C4 = −11/13, FormaSolPart);

$$SolucionParticular := -\frac{3}{13}\cos(3x) - \frac{11}{13}\sin(3x) \quad (29)$$

> SolucionGeneral := y(x) = rhs(SolucionHomogenea) + SolucionParticular

$$SolucionGeneral := y(x) = C1\,e^{x} + C2\,e^{-2x} - \frac{3}{13}\cos(3x) - \frac{11}{13}\sin(3x) \quad (30)$$

>

TERCER REACTIVO

> restart

> Ecuacion := diff(y(x), x$2) − 16· y(x) = 16·x·exp(−4·x)

$$Ecuacion := \frac{d^2}{dx^2}y(x) - 16\,y(x) = 16\,x\,e^{-4x} \quad (31)$$

> EcuacionHomogenea := lhs(Ecuacion) = 0

$$EcuacionHomogenea := \frac{d^2}{dx^2}y(x) - 16\,y(x) = 0 \quad (32)$$

> EcuacionCaracteristica := m··2 − 16 = 0

$$EcuacionCaracteristica := m^2 - 16 = 0 \quad (33)$$

> Raiz := solve(EcuacionCaracteristica)

$$Raiz := 4,\ -4 \quad (34)$$

> $Sol_1 := y(x) = \exp(Raiz_1 \cdot x); Sol_2 := y(x) = \exp(Raiz_2 \cdot x)$

$$Sol_1 := y(x) = e^{4x}$$

$$Sol_2 := y(x) = e^{-4x} \tag{35}$$

> $SolucionHomogenea := y(x) = C1 \cdot rhs(Sol_1) + C2 \cdot rhs(Sol_2)$

$$SolucionHomogenea := y(x) = C1\, e^{4x} + C2\, e^{-4x} \tag{36}$$

> $Q(x) := rhs(Ecuacion)$

$$Q(x) := 16\, x\, e^{-4x} \tag{37}$$

> $Solucion_1 := y(x) = e^{4x}; Solucion_2 := y(x) = e^{-4x}$

$$Solucion_1 := y(x) = e^{4x}$$

$$Solucion_2 := y(x) = e^{-4x} \tag{38}$$

> $with(linalg):$

> $Wrons := Wronskian([rhs(Solucion_1), rhs(Solucion_2)], x);$

$$Wrons := \begin{bmatrix} e^{4x} & e^{-4x} \\ 4\, e^{4x} & -4\, e^{-4x} \end{bmatrix} \tag{39}$$

> $BDet := array([0, Q(x)]);$

$$BDet := \begin{bmatrix} 0 & 16\, x\, e^{-4x} \end{bmatrix} \tag{40}$$

> $SOL := linsolve(Wrons, BDet) : SOL_1; SOL_2;$

$$\frac{2\, e^{-4x}\, x}{e^{4x}}$$

$$-2\, x \tag{41}$$

> $U(t) := int(SOL_1, x); V(t) := int(SOL_2, x)$

$$U(t) := -\frac{1}{32}\, \frac{(1 + 8\, x)\, e^{-4x}}{e^{4x}}$$

$$V(t) := -x^2 \tag{42}$$

> $SolucionParticular := y_p(x) = simplify(U(t) \cdot rhs(Solucion_1) + V(t) \cdot rhs(Solucion_2))$

$$SolucionParticular := y_p(x) = -\frac{1}{32}\, e^{-4x}\, (1 + 8\, x + 32\, x^2) \tag{43}$$

> $SolucionGeneral := y(x) = rhs(SolucionHomogenea) + rhs(SolucionParticular)$

$$SolucionGeneral := y(x) = -\frac{1}{32}\, e^{-4x}\, (1 + 8\, x + 32\, x^2) + C1\, e^{4x} + C2\, e^{-4x} \tag{44}$$

>

CUARTO REACTIVO

> $restart$

> $Ecuacion := diff(y(x), x\$2) + 4 \cdot diff(y(x), x) + 4\, y(x) = 8$

$$Ecuacion := \frac{d^2}{dx^2}\, y(x) + 4 \left(\frac{d}{dx}\, y(x) \right) + 4\, y(x) = 8 \tag{45}$$

> $Variable1 := x_1(t) = y(t)$

$$Variable1 := x_1(t) = y(t) \tag{46}$$

> $Variable := diff\left(x_1(t), t\right) = x_2(t)$

$$Variable := \frac{\mathrm{d}}{\mathrm{d}t}\, x_1(t) = x_2(t) \tag{47}$$

> $DerivVariable2 := diff\left(x_1(t), t\$2\right) = diff\left(x_2(t), t\right)$

$$DerivVariable2 := \frac{\mathrm{d}^2}{\mathrm{d}t^2}\, x_1(t) = \frac{\mathrm{d}}{\mathrm{d}t}\, x_2(t) \tag{48}$$

> $DervDespej := diff(y(t), t\$2) = -4 \cdot diff(y(t), t) - 4\, y(t) + 8$

$$DervDespej := \frac{\mathrm{d}^2}{\mathrm{d}t^2}\, y(t) = -4\left(\frac{\mathrm{d}}{\mathrm{d}t}\, y(t)\right) - 4\, y(t) + 8 \tag{49}$$

>

> $Ecuacion2Sist := diff\left(\, x_1(t), t\right) = -4 \cdot x_2\ (t) - 4 \cdot x_1(t) + 8$

$$Ecuacion2Sist := \frac{\mathrm{d}}{\mathrm{d}t}\, x_1(t) = -4\, x_2\, t - 4\, x_1(t) + 8 \tag{50}$$

>

> $Sistema := Variable, Ecuacion2Sist : Sistema_1; Sistema_2;$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, x_2(t) = v(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, x_1(t) = -4\, x_2\, t - 4\, x_1(t) + 8 \tag{51}$$

>

> *restart*

> $Sistema := diff\left(x_1(t), t\right) = x_2(t), diff\left(x_2(t), t\right) = -4 \cdot x_2(t) - 4 \cdot x_1(t) + 8 : Sistema_1; Sistema_2$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, x_1(t) = x_2(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, x_2(t) = -4\, x_2(t) - 4\, x_1(t) + 8 \tag{52}$$

> $SolucionSistema := dsolve(\{Sistema\})$

$$SolucionSistema := \left\{x_1(t) = 2 + \mathrm{e}^{-2\,t}\,(\_C2 + t\,\_C1),\, x_2(t) = -\mathrm{e}^{-2\,t}\,(2\,\_C2 + 2\,t\,\_C1 - \_C1)\right\} \tag{53}$$

>

QUINTO REACTIVO

>

> *restart*

> $Ecuac := diff(x(t), t\$2) - diff(x(t), t) - 6 \cdot x(t) = 60 \cdot \mathrm{Heaviside}(t - \mathrm{pi})$

$$Ecuac := \frac{\mathrm{d}^2}{\mathrm{d}t^2}\, x(t) - \left(\frac{\mathrm{d}}{\mathrm{d}t}\, x(t)\right) - 6\, x(t) = 60\ \mathrm{Heaviside}(t - \pi) \tag{54}$$

> $Condiciones := x(0) = 0, \mathrm{D}(x)(0) = 0$

$$Condiciones := x(0) = 0, \mathrm{D}(x)(0) = 0 \tag{55}$$

> *with*(*inttrans*);

[*addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace,* **(56)** *invmellin, laplace, mellin, savetable*]

```
> TeLaplEc := subs(Condiciones, laplace(Ecuac, t, s));
```

$$TeLaplEc := s^2\,laplace(x(t), t, s) - s\,laplace(x(t), t, s) - 6\,laplace(x(t), t, s) = 60\,laplace(\mathrm{Heaviside}(t-\pi), t, s) \quad (57)$$

```
> TeLaplSol := subs(simplify(isolate(TeLaplEc, laplace(x(t), t, s))))
```

$$TeLaplSol := laplace(x(t), t, s) = \frac{60\,laplace(\mathrm{Heaviside}(t-\pi), t, s)}{s^2 - s - 6} \quad (58)$$

```
> SolucParticular := invlaplace(TeLaplSol, s, t)
```

$$SolucParticular := x(t) = 2\,\mathrm{Heaviside}(-\pi)\,\left(3\,e^{-2t} - 2\,e^{3t-3\pi} - 3\,e^{-2t+2\pi} + 2\,e^{3t}\right) - 2\,\mathrm{Heaviside}(t-\pi)\,\left(5 - 3\,e^{-2t+2\pi}\right) + 4\,\left(1 - \mathrm{Heaviside}(-t+\pi)\right)\,e^{3t-3\pi} \quad (59)$$

```
>
```

SEXTO REACTIVO

```
> restart
> Ecuacion := diff(y(t), t) =-sin(t) - int(y(tau), tau=0..t)
```

$$Ecuacion := \frac{d}{dt}\,y(t) = -\sin(t) - \left(\int_0^t y(\tau)\,d\tau\right) \quad (60)$$

```
> Condicion := y(0) = 1
```

$$Condicion := y(0) = 1 \quad (61)$$

```
> with(inttrans) :
> TransLapEcuacion := subs(Condicion, laplace(Ecuacion, t, s))
```

$$TransLapEcuacion := s\,laplace(y(t), t, s) - 1 = -\frac{1}{s^2+1} - \frac{laplace(y(t), t, s)}{s} \quad (62)$$

```
> TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))
```

$$TransLapSolucion := laplace(y(t), t, s) = \frac{s^3}{\left(s^2+1\right)^2} \quad (63)$$

```
> SolucionParticular := invlaplace(TransLapSolucion, s, t)
```

$$SolucionParticular := y(t) = \cos(t) - \frac{1}{2}\,t\sin(t) \quad (64)$$

```
>
```

SEPTIMO REACTIVO

```
> restart
> Ecuacion := diff(u(x, y), x$2) - 3·diff(u(x, y), y) = 0
```

$$Ecuacion := \frac{\partial^2}{\partial x^2}\,u(x, y) - 3\,\left(\frac{\partial}{\partial y}\,u(x, y)\right) = 0 \quad (65)$$

```
> EcuacionSeparable := subs(u(x, y) = F(x)·G(t), Ecuacion)
```

$$EcuacionSeparable := \frac{\partial^2}{\partial x^2}\,(F(x)\,G(t)) - 3\,\left(\frac{\partial}{\partial y}\,(F(x)\,G(t))\right) = 0 \quad (66)$$

$$> EcuacionF := \frac{diff(F(x), x\$2)}{F(x)} = 3$$

$$EcuacionF := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 3 \quad (67)$$

> $SolF(x) := dsolve(EcuacionF)$

$$SolF(x) := F(x) = \_C1\, e^{\sqrt{3}\, x} + \_C2\, e^{-\sqrt{3}\, x} \quad (68)$$

> $EcuacionG := \frac{3 \cdot diff(G(t), t)}{G(t)} = 3$

$$EcuacionG := \frac{3 \left(\frac{d}{dt} G(t)\right)}{G(t)} = 3 \quad (69)$$

> $SolG := dsolve(EcuacionG)$

$$SolG := G(t) = \_C1\, e^{t} \quad (70)$$

> $SolGdef := subs(\_C1 = C3, SolG)$

$$SolGdef := G(t) = C3\, e^{t} \quad (71)$$

> $SolucionComp := U(x, y) = rhs(SolF(x)) \cdot rhs(SolGdef)$

$$SolucionComp := U(x, y) = \left(\_C1\, e^{\sqrt{3}\, x} + \_C2\, e^{-\sqrt{3}\, x}\right) C3\, e^{t} \quad (72)$$

> $SolucionCompleta := expand(rhs(SolucionComp))$

$$SolucionCompleta := C3\, e^{t}\, \_C1\, e^{\sqrt{3}\, x} + \frac{C3\, e^{t}\, \_C2}{e^{\sqrt{3}\, x}} \quad (73)$$

>

>