SOLUCIÓN: SEGUNDO EXAMEN FINAL COLEGIADO 2012-1

ESTÁTICA

1.- Equivalencia: Ley de la Gravitación universal con 2ª Ley de Newton

En la superficie

$$G\frac{M_T m_B}{R_T^2} = W_B \Rightarrow G M_T m_B = W_B R_T^2 \quad (1)$$

a la altura H

$$G\frac{M_T m_B}{(R_T + H)^2} = F \quad \text{de}(1) \quad \frac{W_B R_T^2}{(R_T+H)^2} = F$$

$$\Rightarrow H = \left[\sqrt{\frac{W_B}{F}} - 1\right] R_T \; ; \; \underline{H = 1775.13\ km}$$

$\underline{W_B = 981\ kg_f \; ; \; F = 600\ kg_f}$

2.-

a) $\bar{r}_{A/B} = (3, -4, 0)$; $|\bar{r}_{A/B}| = 5$

$\underline{\bar{F} = 6\hat{i} - 8\hat{j}\ N}$

b) Con respecto al eje x

$\bar{r}_{A/E} = (-2, -3, 1)$; $\bar{M}_E = 8\hat{i} + 6\hat{j} + 34\hat{k}\ N\cdot m$

$\underline{\bar{M}_{xx} = (\bar{M}_E \cdot \hat{i})\hat{i} = 8\hat{i}\ N\cdot m}$

Con respecto al eje y

$\bar{r}_{A/C} = (1, -5, 1)$; $\bar{M}_C = 8\hat{i} + 6\hat{j} + 22\hat{k}\ Nm$

$\underline{\bar{M}_{yy} = (\bar{M}_C \cdot \hat{j})\hat{j} = 6\hat{j}\ N\cdot m}$

Con respecto al eje z

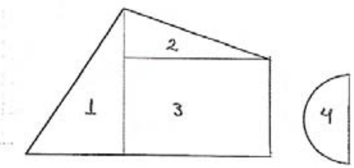
$\bar{r}_{A/D} = (1, -3, 2)$; $\bar{M}_D = 16\hat{i} + 12\hat{j} + 10\hat{k}\ N\cdot m$

$\underline{\bar{M}_{zz} = (\bar{M}_D \cdot \hat{k})\hat{k} = 10\hat{k}\ N\cdot m}$

c) $\underline{\bar{M}_O = \bar{r}_A \times \bar{F} = 8\hat{i} + 6\hat{j} + 10\hat{k}\ N\cdot m}$

$= \bar{M}_{xx} + \bar{M}_{yy} + \bar{M}_{zz}$ ◻

3.-



$\bar{x}_1 = -4\ cm$; $\bar{y}_1 = 0$ $\quad A_1 = 27\ cm^2$

$\bar{x}_2 = 1\ cm$; $\bar{y}_2 = 4$ $\quad A_2 = 13.5\ cm^2$

$\bar{x}_3 = 2.5\ cm$; $\bar{y}_3 = 0$ $\quad A_3 = 54\ cm^2$

$\bar{x}_4 = 7 - \frac{4}{\pi}\ cm$; $\bar{y}_4 = 0$ $\quad A_4 = \frac{9}{2}\pi\ cm^2$

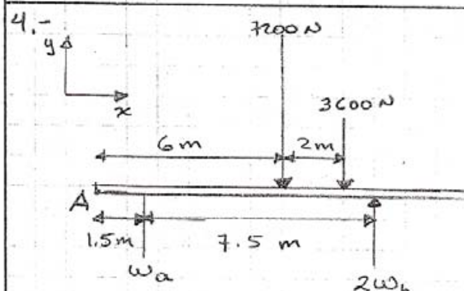
$\Sigma \bar{x}_i A_i = \bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3 - \bar{x}_4 A_4 = -40.46\ cm^3$

$\Sigma \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3 - \bar{y}_4 A_4 = 54\ cm^2$

$\Sigma A_i = A_1 + A_2 + A_3 - A_4 = 80.362\ cm^2$

$\underline{\bar{X} = -0.503\ cm \; ; \; \bar{y} = 0.672\ cm}$

4.-



$\Sigma F = \omega_a + 2\omega_b - 10800 = 0$

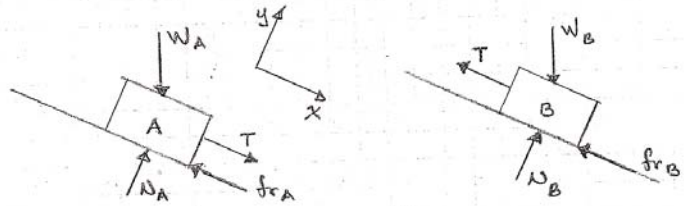
$\Sigma M_A = 1.5\omega_a + 9(2\omega_b) - 6(7200) - 8(3600) = 0$

$$\begin{cases} \omega_a + 2\omega_b = 10800 \\ 1.5\omega_a + 18\omega_b = 72000 \end{cases}$$

$\underline{\omega_a = 3360\ N/m}$

$\underline{\omega_b = 3720\ N/m}$

5.-



Bloque A

$\Sigma F_x = T + W_A \operatorname{sen}\theta - fr_A = 0 \quad \ldots (1)$

$\Sigma F_y = N_A - W_A \cos\theta = 0$

$\rightarrow N_A = W_A \cos\theta$

Bloque B

$\Sigma F_x = -T + W_B \operatorname{sen}\theta - fr_B = 0 \quad \ldots (2)$

$\Sigma F_y = N_B - W_B \cos\theta = 0$

$\rightarrow N_B = W_B \cos\theta$

En el límite $fr = \mu_s N$

sustituyendo en (1) y (2)

(1) $\rightarrow T + W_A \operatorname{sen}\theta - \mu_A W_A \cos\theta = 0$

(2) $\rightarrow -T + W_B \operatorname{sen}\theta - \mu_B W_B \cos\theta = 0$

$$\tan\theta = \frac{\mu_A W_A + \mu_B W_B}{W_A + W_B}$$

$\mu_A = 0.35 \ ; \ W_A = 60\ kg_f$

$\mu_B = 0.20 \ ; \ W_B = 100\ kg_f$

$\underline{\theta = 14.37°}$

Por otro lado, los ángulo de reposo para los bloques sin la cuerda son:

$\theta_A = 19.29°$ y $\theta_B = 11.31°$ de $\theta = \operatorname{ang}\tan(\mu_s)$

por lo que, el resultado es correcto.