Solución: Segundo Examen Final Colegiado 2013-2

Estática

1) a) $\vec{R} = 25\left(-\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k}\right) \rightarrow \vec{R} = -20\hat{j} - 15\hat{k}$ N ;

$\Rightarrow \vec{F}_2 = \vec{R} - \vec{F}_1 \rightarrow \underline{\vec{F}_2 = -7\hat{i} - 21\hat{j} - 10\hat{k} \text{ N}}$

b) Un punto sobre la línea de acción de $\vec{R}$ es $P(1, 4, 3)$

y con $Q(x_0, y_0, 0) \rightarrow \vec{r} = (1-x_0,\ 4-y_0,\ 3)$

Con respecto al origen $\vec{M}_0 = \vec{r}\times\vec{R} = 5[3y_0\hat{i} + (3-3x_0)\hat{j} - 4(1-x_0)\hat{k}]$

y $\vec{M}_0 = 15\hat{j} - 20\hat{k}$

por lo tanto $\underline{x_0 = 0}$ y $\underline{y_0 = 0}$

2) $\vec{F}_1 = -F_1\hat{i}$, $\vec{F}_2 = F_2\hat{j}$, $\vec{F}_3 = -F_3\hat{k}$, $\vec{F}_4 = \frac{315}{0.63}(0.24\hat{i} - 0.12\hat{j} - 0.57\hat{k})$

$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$; $\vec{R} = (120 - F_1)\hat{i} + (F_2 - 60)\hat{j} - (285 + F_3)\hat{k}$ N

y $\vec{R} = -570\hat{k}$ N $\Rightarrow$ $\underline{F_1 = 120 \text{ N}}$, $\underline{F_2 = 60 \text{ N}}$; $\underline{F_3 = 285 \text{ N}}$

Con respecto al origen

$\vec{M}_1 = \vec{r}_1\times\vec{F}_1 = \vec{0}$; $\vec{M}_2 = \vec{r}_2\times\vec{F}_2 = (1\hat{i} + 0.5\hat{j})\times(60\hat{j}) = 60\hat{k}$ N·m

$\vec{M}_3 = \vec{r}_3\times\vec{F}_3 = (1\hat{i})\times(-285\hat{k}) = 285\hat{j}$ N·m

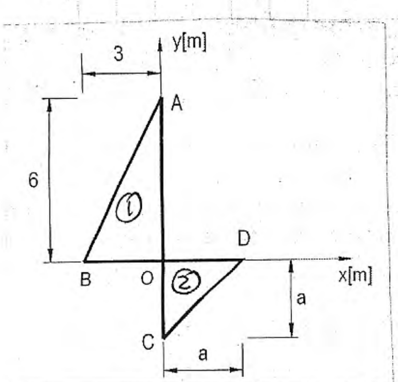
$\vec{M}_4 = \vec{r}_4\times\vec{F}_4 = (0.5\hat{j})\times(120\hat{i} - 60\hat{j} - 285\hat{k}) = -142.5\hat{i} - 60\hat{k}$ N·m

$\vec{M}_0 = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4$; $\vec{M}_0 = -142.5\hat{i} + 285\hat{j}$ N·m ... ①

al reducirse el sistema a una sola fuerza $\vec{M}_0 = (x, y, 0)\times\vec{R}$

$\vec{M}_0 = -570y\hat{i} + 570x\hat{j}$ N·m igualando con ① $\underline{x = 0.5 \text{ m}}$, $\underline{y = 0.25 \text{ m}}$

3)



$A_1 = 9 \text{ m}^2$; $\bar{x}_1 = -1$ m ; $\bar{y}_1 = 2$ m ; $A_2 = \frac{a^2}{2}$ m², $\bar{x}_2 = \frac{a}{3}$ m, $\bar{y}_2 = \frac{a}{3}$ m

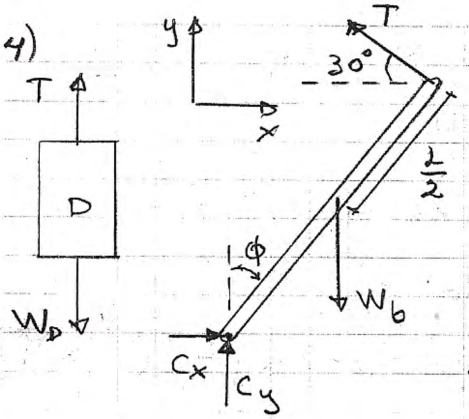
Para quedar en ①

$$\bar{x} = \frac{(-1)9 + \frac{a}{3}\left(\frac{a^2}{2}\right)}{9 + \frac{a^2}{2}} = \frac{-9 + \frac{a^3}{6}}{9 + \frac{a^2}{2}} \rightarrow \frac{a^3}{6} - 9 < 0 \Rightarrow a < \sqrt[3]{54} \text{ m}$$

$$\bar{y} = \frac{2(9) + \left(-\frac{a}{3}\right)\left(\frac{a^2}{2}\right)}{9 + \frac{a^2}{2}} = \frac{18 - \frac{a^3}{6}}{9 + \frac{a^2}{2}} \Rightarrow 18 - \frac{a^3}{6} > 0 \Rightarrow a < \sqrt[3]{108} \text{ m}$$

El rango de valores es: $a \in (0, \sqrt[3]{54})$ m

4)



Bloque $\Sigma F_y = T - W_D = 0 \qquad \underline{T = W_D}$

Barra

$$\Sigma F_x = C_x - \frac{\sqrt{3}}{2}T = 0$$

$$\Sigma F_y = C_y + \frac{1}{2}T - W_b = 0$$

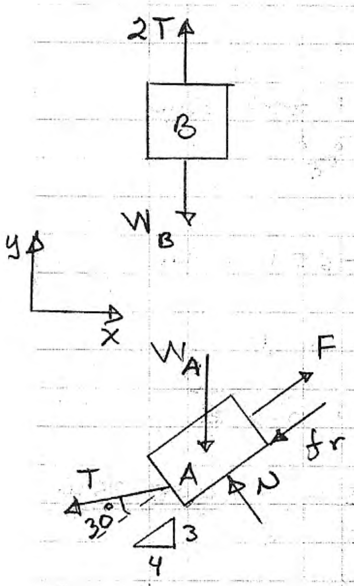
$$\Sigma M_C = -\frac{L}{2}\left(\frac{3}{5}\right)W_b + \frac{4}{5}L\left(\frac{\sqrt{3}}{2}T\right) + \frac{3}{5}L\left(\frac{1}{2}T\right) = 0$$

$$\Rightarrow T = \left(\frac{3}{4\sqrt{3}+3}\right)W_b$$

$\Rightarrow \underline{T = 30.2\ N}$, $C_x = 26.17\ N$, $C_y = 84.9\ N$, $\underline{C = 88.84\ N}$

$\underline{W_D = 30.2\ N}$

5)



Bloque B

$$\Sigma F_y = 2T - W_B = 0$$

$$T = \frac{W_B}{2}$$

Bloque A

$$\Sigma F_x = F - \frac{3}{5}W_A - \frac{\sqrt{3}}{2}T - fr = 0$$

$$\Sigma F_y = N - \frac{4}{5}W_A + \frac{1}{2}T = 0 \ ; \ N = \frac{4}{5}W_A - \frac{1}{2}\left(\frac{W_B}{2}\right)$$

$fr = \mu_s N$ (movimiento inminente)

y de ΣF_x

$$F = \left(\frac{3}{5} + \frac{4}{5}\mu_s\right)W_A + \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\mu_s\right)W_B$$

$$F = W_A + 0.31\,W_B$$

$\underline{F = 29.24\ N}$