Universidad Nacional Autónoma de México
Facultad de Ingeniería
Cálculo Vectorial
Solución Primer Examen Extraordinario
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1. $f(x,y) = (x^2 + y^2)e^{-x}$

$$f(x,y) = x^2e^{-x} + y^2e^{-x}$$

$$\frac{\partial f}{\partial x} = -x^2e^{-x} + 2xe^{-x} + y^2e^{-x} = 0........(1)$$

$$\frac{\partial f}{\partial y} = 2y^2e^{-x} = 0$$

$$e^{-x} \neq 0 \qquad 2y = 0$$

$$y = 0$$

$De\ (1)$

$$e^{-x}(2x - x^2) = 0$$

$$x(2-x) = 0 \begin{cases} x = 0 \\ x = 2 \end{cases}$$

$$P_{C1} = (0,0) \qquad P_{C2} = (2,0)$$

$$\Delta_H = \begin{vmatrix} x^2e^{-x} - 2xe^{-x} - 2xe^{-x} + 2e^{-x} + y^2e^{-x} & -2ye^{-x} \\ -2ye^{-x} & 2e^{-x} \end{vmatrix}$$

$$\Delta_H\Big|_{P_{C1}} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0 \qquad f_{xx} = 2 > 0 \qquad \boxed{\therefore P_{C1}\ es\ m_R}$$

$$\Delta_H\Big|_{P_{C2}} = \begin{vmatrix} 4e^{-2} - 8e^{-2} + 2e^{-2} & 0 \\ 0 & 2e^{-2} \end{vmatrix} = -2e^{-2}(2e^{-2}) < 0 \qquad \boxed{\therefore P_{C2}\ es\ punto\ silla}$$

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15 PUNTOS

2. $\overline{F}(x, y, z) = \left(\dfrac{y}{e^{2x}}\right) i + \ln\left(\dfrac{yz}{x^3}\right) j + \left(\sqrt{y^2 z}\right) k$

$$\overline{F} = \left(ye^{-2x}, \quad \ln(y) + \ln(z) - 3\ln(x) , \quad y\sqrt{z} \right)$$

$$\overrightarrow{\nabla} \bullet \overrightarrow{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = -2ye^{-2x} + \frac{1}{y} + \frac{y}{2\sqrt{z}}$$

$$\boxed{\overrightarrow{\nabla}\left(\overrightarrow{\nabla} \bullet \overrightarrow{F}\right) = \left(4ye^{-2x}\right)i + \left(-2e^{-2x} - \frac{1}{y^2} + \frac{1}{2\sqrt{z}}\right)j + \left(-\frac{1}{4}\frac{y}{\sqrt{z^3}}\right)k}$$

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3.

$$\overline{F}(r,\theta,z) = -\frac{z}{r^2}\sec^2\theta\, \overline{e}_r + \frac{2z}{r^2}\sec^2\theta \tan\theta\, \overline{e}_\theta + \left(\frac{1}{r} + \frac{1}{r}\tan^2\theta\right)\overline{e}_z$$

$$rot\overline{F} = \frac{1}{r}\begin{vmatrix} \overline{e}_r & r\overline{e}_\theta & \overline{e}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ -\dfrac{z}{r^2}\sec^2\theta & \dfrac{2z}{r}\sec^2\theta\tan\theta & \dfrac{1}{r} + \dfrac{\tan^2\theta}{r} \end{vmatrix}$$

$$= \frac{1}{r}\left[\frac{2\tan\theta\sec^2\theta}{r} - \frac{2\sec^2\theta\tan\theta}{r}, r\left(-\frac{1}{r^2} - \frac{\tan^2\theta}{r^2} + \frac{\sec^2\theta}{r^2}\right), -\frac{2z}{r^2}\sec^2\theta\tan\theta + \frac{2z}{r^2}\sec^2\theta\tan\theta\right]$$

$$= [0,0,0] \qquad \therefore \overline{F}\ es\ conservativa$$

$$\therefore \quad \phi = \int -\frac{z}{r^2}\sec^2\theta dr \cup \int \frac{2z}{r}\sec^2\theta\tan\theta d\theta \cup \int\left(\frac{1}{r} + \frac{\tan^2\theta}{r}\right)dz$$

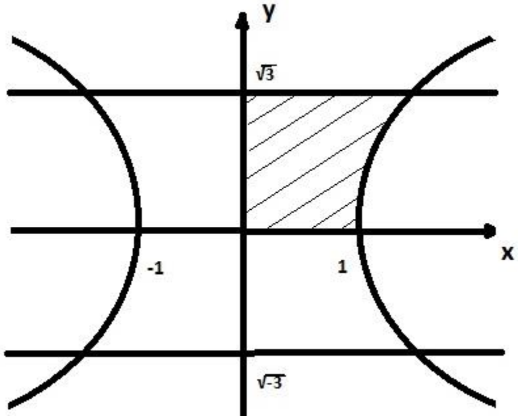
$$= \frac{z}{r}\sec^2\theta \cup \frac{z}{r}\tan^2\theta \cup \frac{z}{r} + \frac{z}{r}\tan^2\theta$$

$$= \frac{z}{r}\sec^2\theta + \frac{z}{r}\tan^2\theta + \frac{z}{r}\Bigg|_{A(1,\frac{\pi}{4},1)}^{B(2,0,2)} = 1 - (2+1) = \boxed{-2\ u.t.}$$

15 PUNTOS

4.

$$y=-\sqrt{3} \quad , \quad y=\sqrt{3} \quad , \quad x^2-y^2=1 \quad , \quad x+y+z=2 \quad , \quad z=0$$



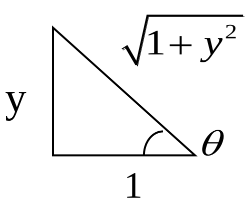
$$x=\pm\sqrt{1+y^2}$$

$$R=\left\{(x,y)\middle|-\sqrt{1+y^2}\le x\le\sqrt{1+y^2},-\sqrt{3}\le x\le\sqrt{3}\right\}$$

$$V=4\int_0^{\sqrt{3}}\int_0^{\sqrt{1+y^2}}(2-x-y)\,dx\,dy=4\int_0^{\sqrt{3}}2x-\frac{x^2}{2}-xy\Big|_0^{\sqrt{1+y^2}}dy$$

$$=4\int_0^{\sqrt{3}}\left(2\sqrt{1+y^2}-\frac{(1+y^2)}{2}-y\sqrt{1+y^2}\right)dy \ \ldots(1)$$

$$I=\int\sqrt{1+y^2}dy=\int\sec\theta\sec^2\theta\,d\theta=\sec\theta\tan\theta-\int\sec\theta\left(\sec^2\theta-1\right)d\theta$$



$$\tan\theta=y \qquad u=\sec\theta \qquad dv=\sec^2\theta d\theta$$

$$\sec\theta=\sqrt{1+y^2} \qquad du=\sec\theta\tan\theta d\theta \qquad v=\tan\theta$$

$$2I=\sec\theta\tan\theta+\ln\left(\sec\theta+\tan\theta\right)$$

$$I=\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln\left(\sec\theta+\tan\theta\right)$$

Sustituyendo en (1)

$$V=4\left[y\sqrt{1+y^2}+\ln\left(\sqrt{1+y^2}+y\right)-\frac{1}{2}y-\frac{y^3}{6}-\frac{1}{3}\left(\sqrt{1+y^2}\right)^3\right]_0^{\sqrt{3}}$$

$$=4\left[\left(2\sqrt{3}+\ln\left(2+\sqrt{3}\right)-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}-\frac{8}{3}\right)-\left(-\frac{1}{3}\right)\right]$$

$$V=4\left[\sqrt{3}+\ln\left(2+\sqrt{3}\right)-\frac{7}{3}\right]\left[u^3\right]$$

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$$a)\quad \left.\begin{array}{l}\overline{\nabla} u=(1,1)\\ \overline{\nabla} v=(2,-1)\end{array}\right\}\overline{\nabla} u\cdot\overline{\nabla} v=(1,1)\cdot(2,-1)=2-1=1\neq 0$$

$$\therefore \textit{No es ortogonal}$$

$$b)\quad u+v=3x \quad \therefore \quad x=\frac{u+v}{3}$$

$$v=2\left(\frac{u+v}{3}\right)-y \quad\longrightarrow\quad y=\frac{2u}{3}+\frac{2v}{3}-v \quad\longrightarrow\quad y=\frac{2u-v}{3}$$

$$\overline{r}(u,v)=\left(\frac{u+v}{3},\frac{2u-v}{3}\right)$$

$$\overline{r_u}=\left(\frac{1}{3},\frac{2}{3}\right) \;\longrightarrow\; h_u=\left|\overline{r_u}\right|=\sqrt{\frac{5}{9}}=\frac{\sqrt{5}}{3}$$

$$\overline{r_v}=\left(\frac{1}{3},-\frac{1}{3}\right) \;\longrightarrow\; h_v=\left|\overline{r_v}\right|=\sqrt{\frac{2}{9}}=\frac{\sqrt{2}}{3}$$

$$c)\quad \overline{e_u}=\frac{\overline{r_u}}{h_v}=\frac{\left(\frac{1}{3},\frac{2}{3}\right)}{\frac{\sqrt{5}}{3}}=\frac{(1,2)}{\sqrt{5}}$$

$$\overline{e_v}=\frac{\overline{r_v}}{h_v}=\frac{(1,-1)}{\sqrt{2}}$$

$$d)\quad J\left(\frac{x,y}{u,v}\right)=\begin{vmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{2}{3} & -\frac{1}{3}\end{vmatrix}=-\frac{1}{9}-\frac{2}{9}=-\frac{1}{3}$$

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$$\overline{F}(x,y,z) = (\mathrm{x})\,\mathrm{i} + (\mathrm{y})\,\mathrm{j} + (\mathrm{z})\,\mathrm{k}$$

$$\mathrm{z} = \sqrt{x^2 + y^2} \qquad y \qquad z = 2$$

$$Flujo = \iint_S \overline{F} \cdot \overline{n}\, ds = \iiint_D div\overline{F}\, dv$$

$$div\overline{F} = 1 + 1 + 1 = 3$$

$$\iiint_D div\overline{F}\, dv = \iiint_D 3\, dv = 3\, volumen\, D$$

$$z = \sqrt{x^2 + y^2} \rightarrow x^2 + y^2 = 4 \rightarrow r = 2$$

$$h = 2$$

$$3\, volumen\, D = 3\left(\frac{1}{3}\pi(2^2)(2)\right) = 8\pi \ u.\ de\ flujo$$

15 PUNTOS